**Mathematics Methods**

**Units 1&2**

**Investigation 3**

**Pascal’s Triangle**

**50 minutes 40 marks Name:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Answer all questions in the spaces provided. Show all necessary working. Calculators allowed.**

Pascal’s Triangle is a triangular array of numbers, named after French mathematician Blaise Pascal.

Sum

Row 0 1 \_\_\_\_

Row 1 1 1 \_\_\_\_

Row 2 1 2 1 \_\_\_\_

Row 3 1 3 3 1 \_\_\_\_

Row 4 1 4 6 4 1 \_\_\_\_

Row 5 1 5 10 10 5 1 \_\_\_\_

Row 6 1 6 15 20 15 6 1 \_\_\_\_

Row 7

Row 8

Row 9

**Pascal’s Triangle Name:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

1. [2, 3 = 5 marks]

(a) Explain how the numbers in each row are obtained from the row above it.

(b) Complete the next three rows of the triangle, in the space provided.

2. [1, 1, 1, = 3 marks]

Using the triangle, and extending the evident patterns, answer the following questions:

(a) How many terms are there in row 12?

(b) What are the first two terms in row 15?

(c) What are the last two terms in row 25?

3. [3, 2, 2, 1 = 7 marks]

1. What is the sum of the terms in each of the first 7 rows of the triangle?

(You should write your answers in the spaces provided next to the triangle.)

1. What is the sum of the terms in row 14?
2. What is the sum of the terms in row 8 ÷ sum of the terms in row 5?
3. What is the sum of the terms in row n?

4. [4, 2, 3, 3, 2 = 14 marks]

(a) There are 5 books on a shelf. Complete the table below to find the number of ways you can choose

0, 1, 2, 3, 4 or 5 books.

|  |  |  |
| --- | --- | --- |
| Choose | Possibilities | Number of ways |
| 0 books | - | 1 |
| 1 book | A, B, C, D, E | 5 |
| 2 books | AB, AC, AD, AE,  BC, BD, BE  CD, CE  DE |  |
| 3 books | ABC, ABD, ABE, ACD, ACE,  ADE,  BCD, BCE  BDE,  CDE |  |
| 4 books |  |  |
| 5 books |  |  |

(b) What is the connection between the number of ways to choose 5 books, and the entries in Pascal’s triangle?

(c) Show that the same relationship works for choosing 0, 1, 2, 3 or 4 books from a shelf

|  |  |  |
| --- | --- | --- |
| Choose | Possibilities | Number of ways |
| 0 books | - | 1 |
| 1 book | A, B, C, D | 4 |
|  |  |  |
|  |  |  |
|  |  |  |

(d) By using the appropriate row in Pascal’s Triangle, find out how many ways there are to choose:

(i) 3 books from a shelf with 6 books

(ii) 4 books from a shelf with 8 books

(iii) 7 books from a shelf with 9 books

1. Which is greater: the number of ways of choosing 5 players from a squad of 8 or the number

of ways of choosing 3 cards from a set of 9 cards?

5. [4, 3, 1, 3 = 11 marks]

*Binomial expansions* are expansions of the form . (‘Binomial’ because there are two terms in the bracket).

(a + b)0 = 1

(a + b)1 = a + b

(a + b) = (a + b)(a + b)

= a + 2ab + b

(a + b) = (a + b)(a + b)(a + b)

= (a+ 2ab + b)(a + b)

= a+ 2ab + ab+ ab + 2ab+ b

= a+ 3ab + 3ab+ b3

(a + b)4 = a4 + 4a3b + 6a2b2 + 4ab3 + b4 (If you want to be convinced of this, expand it on your calculator!!)

(a) Reading the terms in each expansion from **left** to **right**:

(i) Describe what is happening to the power of the ‘a’ value.

(ii) Describe what is happening to the power of the ‘b’ value.

(iii) Looking at the COEFFICIENT of each term (the number in front of each term), describe the pattern that you see. (Hint: list the coefficients for each of the 5 binomial expansions shown above)

(b) Use the patterns you have developed in part (a) to write down the binomial expansion for

(a + b)6

(c) State the third term of the binomial expansion for (a + b)8

(d) Using the patterns from above, find the expansion of